

A comparative experimental study for dry and wet collisions

F-C Ciornei¹, S Alaci¹, P Bulai¹ and I-C Romanu¹

¹Mechanics and Technologies Department, „Stefan cel Mare” University of Suceava, Suceava, Romania

E-mail: florina.ciornei@usm.ro

Abstract. The collision between polyurethane prismatic foam and a dropping steel ball is investigated experimentally. Because the foam presents large deformations, an analytical model is not available. The impact is studied on a test rig from laboratory, for dry foam and for the mineral oil wetted foam. The post impact velocity is found measuring the flight time and the coefficient of restitution and lost energy are estimated. For the dry impact case the damping effect is smaller than for the case of oil-wetted target.

1. Introduction

The mathematical models that describe the behaviour of a system with percussions have continuously evolved. A characteristic of these systems is the sudden variation of the kinematical parameters of first order without any change in the system's configuration.

The simplest models study systems made of two bodies with unidirectional motion and accept as hypothesis that percussion is an instantaneous phenomenon [1-3]. In order to describe the change of the kinematical state of the system, a series of coefficients, expressing the ratio between the values of the considered parameter post and before impact, are used. Amid these parameters, the most important is the coefficient of restitution, defined by Newton as the ratio with changed sign between the postimpact relative velocity and the relative velocity before impact:

$$e = -\frac{v'_2 - v'_1}{v_2 - v_1} \quad (1)$$

According to equation (1), the values of the coefficient of restitution may vary from $e=0$ which is characteristic to plastic collision, when due to impact the system loses the entire kinetic energy, to $e=1$ which is the case of elastic collision, when the kinetic energy of the system is maintained. The requirement of modelling more complex dynamical systems lead to friction consideration, as shown in [4]. Kane [5] analysed the impact with friction for a pendulum and using the equation (1) for the coefficient of restitution reached a paradoxical conclusion, that the final energy of the system is greater than the initial one. To overcome this situation, it is necessary to modify the definition of the coefficient of restitution, equation (1). To this purpose, the notion of percussion is introduced:

$$P = \int_0^t F(t) dt \quad (2)$$

where F is the impact force between the two bodies and t is the time variable. It is accepted that the collision period $[0, t_f]$, is divided in two phases: the compression, during $[0, t_c]$, where t_c is the instant corresponding to the maximum approach between the bodies and the restitution phase, during

the interval $[t_c, t_f]$, where t_f is the final time, when the bodies separate. According to [6], Poisson introduced the following expression for the coefficient of restitution:

$$e = \frac{\mathbf{n} \cdot \int_{t_c}^{t_f} \mathbf{F} dt}{\mathbf{n} \cdot \int_0^{t_c} \mathbf{F} dt} = \frac{P_{nr}}{P_{nc}} \quad (3)$$

where $\mathbf{n}$ is the versor of the normal to the surfaces in the theoretical point of contact. Thus, the coefficient of restitution is the ration between the normal percussion from the restitution phase and the normal percussion from the compression phase.

Wang and Mason elaborated a classification of plane percussions with dry friction and made a foremost observation as conclusion, that is, unlike forces, where the presence of friction forces requires the existence of normal forces, the tangential percussions may exist independent of the normal percussions. Based on this remark, Wang and Mason [1] emphasize that when studying a problem of impact with friction, the principle of linkages formulated by Kilmister and Reeve [7] must be considered; *the constraints should be maintained using forces as long as possible and only on contrary situation, the percussions should be used.*

One of the main drawbacks of the instantaneous impact hypothesis consists in the impossibility of estimation of the contact forces that develop during the impact process. The requirement of correct design of the regions of different mechanical parts coming into contact compelled the decline of instantaneous collision and the acceptance of the hypothesis of finite duration collision. Timoshenko [Ti] considers the perfect elastic impact between two spheres under the assumption of Hertz type impact forces and determines the values of the maximum impact force and of the total collision time.

The centric impact of two viscoelastic spheres was analysed by Lankarani [9] who concluded that the only equation provided by the dynamic theorems [10] is the momentum theorem. To solve the problem, Lankarani used an energy hypothesis and accepts that the quantities of energy lost by friction during the two phases of impact, compression and restitution, are equal and describes the impact force using the equation:

$$F = Kx^\alpha \left[1 + \frac{3(1-e)^2}{4} \frac{\dot{x}}{v_0} \right] \quad (4)$$

where K is a coefficient depending on the elastic and geometrical characteristics of the bodies [11], the exponent takes the value $\alpha = 3/2$ for a point Hertz contact type and v_0 is the initial relative velocity. The model proposed by Lankarani describes accurately the quasielastic collisions for which $e > 0.9$. Flores, [12-13], makes the assumption that the characteristic point describes arcs of ellipse in the phase plane and improves the Lankarani model proposing the following equation for the impact force:

$$F = Kx^\alpha \left[1 + \frac{8(1-e)}{5e} \frac{\dot{x}}{v_0} \right] \quad (5)$$

The equation (5) can be applied for any type of collision, regardless of the values of the coefficient of restitution.

As it can be observed form equation (4) and (5), even for describing the behaviour of the simplest systems, nonlinear differential equations appear. It is expected that when considering more complex systems, with the aim of an accurate description of their behaviour, more intricate differential equations will be used. Additionally, the equations describing the models may lead to the bifurcation phenomenon that makes impossible reaching a conclusion regarding the evolution of the system for a longer period.

2. Statement of the problem and proposed solution

For the situations mentioned above, one of the solutions is to construct the actual device and to elaborate the conclusions based on an experimental study. In this category is framed the present paper that studies the effect the moist upon the damping characteristics of a material. The bodies used in the experiment are a bearing ball as the projectile and a polyurethane parallelepiped, the target. The experiments were conducted in two situations: first, for dry polyurethane foam and second, for wet polyurethane obtained after the foam was soaked in mineral oil and then squeezed. A theoretical model is difficult to elaborate for lubricant-wet body since even for the dry body, none of the equations (4) or (5) is applicable because the exponent α takes different values for the compression and restitution phase [14] and moreover, the hypothesis of small deformations required by the Hertzian theory is not valid. The principle scheme of the test rig is presented in figure 1.

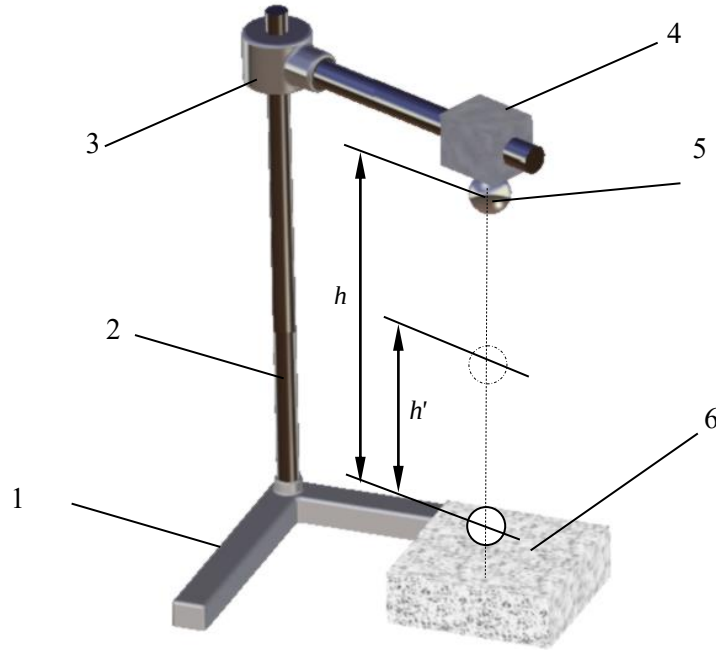


Figure 1. The principle scheme of the test rig

The base 1 supports the cylindrical vertical rod 2. The part 3 can glide along the rod 2 and it can be blocked with a screw in the needed position. An horizontal rod is fixed to the part 3 and a prismatic body 4 can slide on it; the body 4 is set on the required position with a screw. A conical hole is made in the inferior side of the part 4. The ball 5 is brought into contact to the conical hole and thus the same launch position h is ensured. When the ball is set free, it falls and collides the prismatic polyurethane foam 6 and rebounds up the height h' .

The velocity of the ball when it collides the surface of the polyurethane foam is:

$$v = \sqrt{2gh} \quad (6)$$

The post impact velocity of the ball is:

$$v' = \sqrt{2gh'} \quad (7)$$

The distance h from equation 6 is well determined because is constructive parameter of the test rig, but the height h' is difficult to determine. To carry out this, the experiment is filmed with a camera at 240 frames / sec . The film is split into frames and the precise duration t' of the bounce of the ball between the first and the second impact can be found. During this period, the ball reaches the height h' and then falls and impacts the target again:

$$v' = g \frac{t'}{2} \quad (8)$$

With known values of the velocity before and after impact, the coefficient of restitution and the energy lost during collision can be found.

3. Experimental results. Discussions

A question that was raised during the experimental tests concerns the estimation of the flight time of the ball, specifically the instant when the flight time t' starts. As it can be observed from figure 2, during the impact period, the polyurethane target presents large deformation, comparable to the dimensions of the ball, and it also moves. A simple calculus shows that the ball is in contact with the target for a duration of $(1581 - 1573) / 240 = 0.033 \text{ sec}$. The polyurethane is free on the board of the test-rig and because there is possible that it will deform differently for two successive launchings, the time of the flight was calculated between the instant when the ball reaches the target for the first time and the instant when it detaches completely from the target, after the second impact.

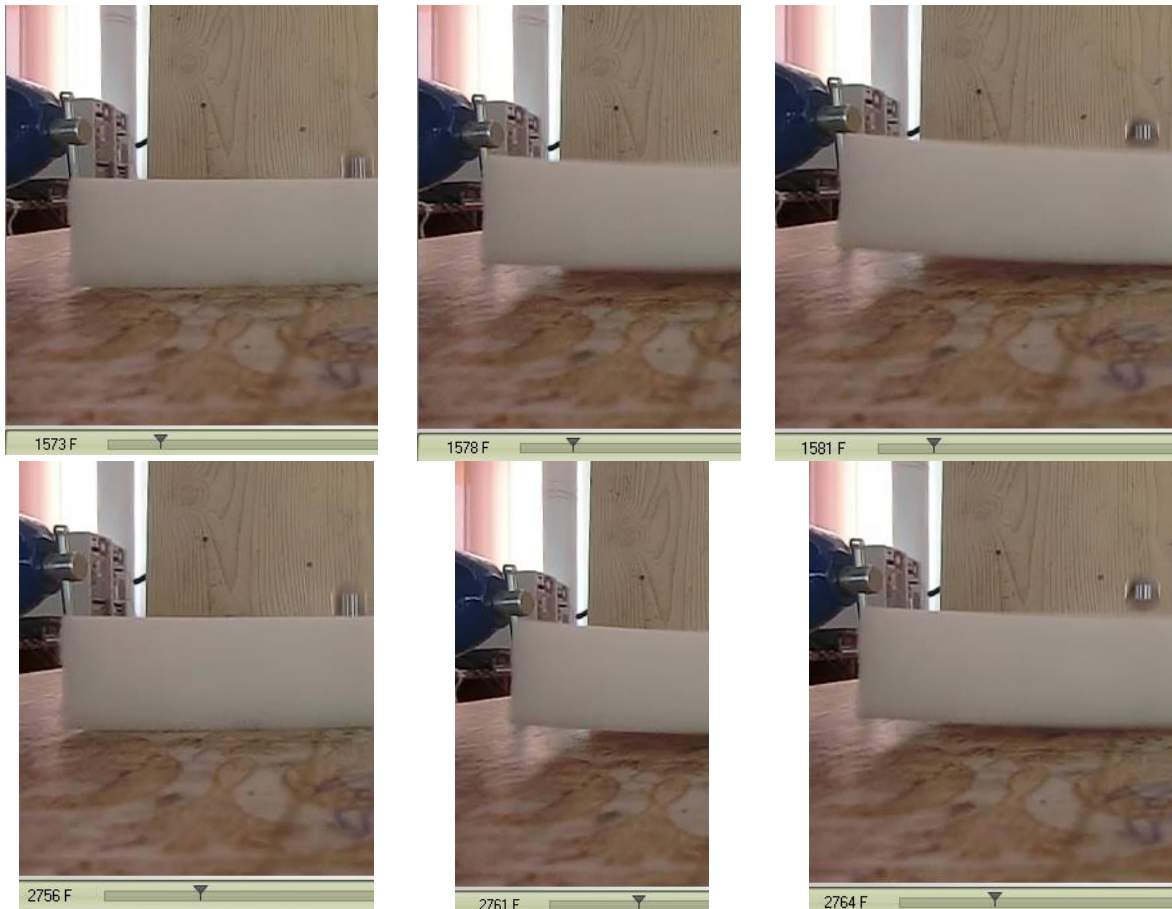


Figure 2. Images of the ball during the contact period for two successive launchings

In figures 3 and 4 there are presented two film frames with the ball at the moment when it detaches from the surface of the target and at the moment when it contacts again the foam after the flight time.



Figure 3. The ball at the first impact with the target



Figure 4. The ball at the second impact with the target

From each figure, in the lower left side it can be noticed the index of the frame that permits calculation of the time of flight:

$$t' = \frac{f_{new} - f_{old}}{240 \frac{fr}{sec}} = \frac{1663 - 1573}{240} sec = 0.375 sec \quad (9)$$

For each of the two cases, dry and wet foam, 12 launches were made. The results are presented in figure 5, where there are also shown the mean values of the flight time for the launchings: $t'_{dry} = 0.376 sec$ and $t_{wet} = 0.365$.

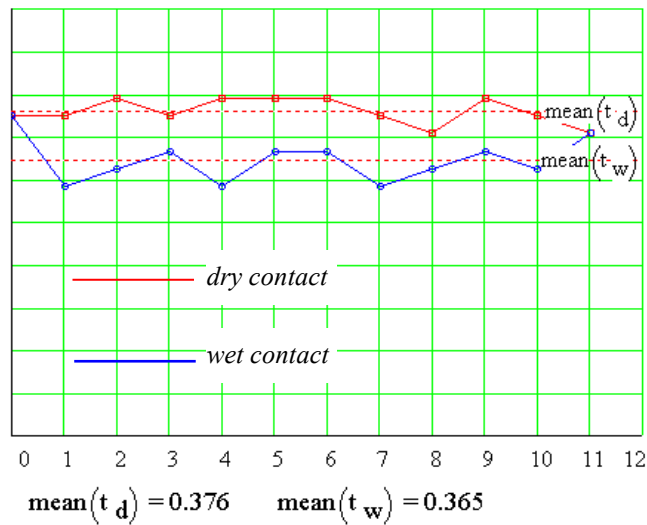


Figure 5. The values of the time of flight for the two cases

It is observed that the time of flight of the ball after the impact with the oil-wet foam is smaller and from here, the conclusion that the post impact velocity of the ball is smaller for the oil-wet contact than for the dry contact.

In the experiments made, the launching height of the ball was $h=0.38m$ and thus the impact velocity with the target, according to equation (6), is $v=2.73m/sec$. Based on the relation (8), the post impact velocities are:

$$v'_{dry}=1.844m/sec, \quad v'_{wet}=1.788 \quad (10)$$

And the corresponding coefficients of restitution are:

$$e_{wet}=0.655, \quad e_{dry}=0.676 \quad (11)$$

The ratio of the energies lost during impact is:

$$\frac{\Delta E_{c_wet}}{\Delta E_{c_dry}} = \frac{\frac{mv^2}{2} - \frac{mv_{wet}^2}{2}}{\frac{mv^2}{2} - \frac{mv_{dry}^2}{2}} = \frac{1 - \frac{v_{wet}^2}{v^2}}{1 - \frac{v_{dry}^2}{v^2}} = \frac{1 - e_{wet}^2}{1 - e_{dry}^2} = 1.05 \quad (12)$$

In order to estimate the magnitude of the maximum impact force, it is accepted that during the contact period, the impact force has a variation of the form:

$$F(t) = F_0 \left[1 - \cos \frac{2\pi t}{t_f} \right] \quad (13)$$

where t_f is the total contact time, $t_f=0.033sec$. The impact force is plotted in figure 6.

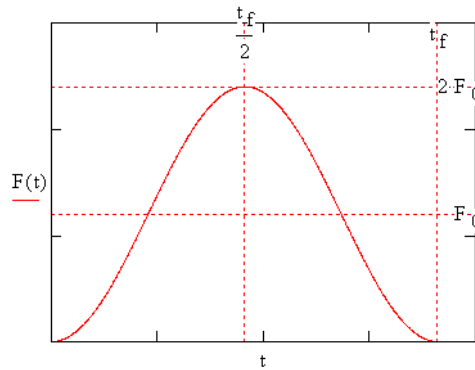


Figure 6. The accepted variation of the impact force

The mean value of the impact force during the collision is:

$$F_{med} = \frac{\int_0^{t_f} F(t) dt}{t_f} = F_0 \quad (14)$$

On the other side, the mean impact force can be calculated using the relation:

$$F_{med} = \frac{m(v - v')}{t_f} \quad (15)$$

Since for the accepted variation of the force the maximum force is:

$$F_{max} = 2F_0 \quad (16)$$

It results

$$\frac{F_{max}}{2} = \frac{m(v - v')}{t_f} \quad (17)$$

The relation (17) permits an estimation of the maximum impact force:

$$F_{max} = \frac{2m(v - v')}{t_f} \quad (18)$$

The ball used in the experiments has the diameter $d = 3/4'' = 19mm$ and the mass $m = 0.028kg$. Applying the relation (18) for the dry impact, for which there were determined $v = 2.73$ m/s, $v' = 1,844$ m/s, it is obtained $F_{max} \approx 1.5newton$.

In literature, [15-17], tests performed on porous materials or on materials capable of absorbing liquids, conclude that the wet materials behave considerably different from the dry ones both in quasi-static and dynamic tests: the stiffness, strength and the elasticity modulus decrease but the lost energy increases as more liquid is absorbed by the material. This is in agreement with the results of the present work, given that the damping for the tests on oil-wet foam was greater than for the dry case.

4. Conclusions

The paper presents an experimental study concerning energy aspects for the impact between a ball and a polyurethane target, for two situations: the dry polyurethane and the oil-wet polyurethane.

The test rig was made in the laboratory and ensures the same conditions for ball launching. The impact velocity is found by calculus, knowing the height from which the ball falls and the post impact velocity is found by using a video camera, based on the duration of the post impact flight time. During the tests and recordings it was noticed that the deformations of the target are comparable to the dimensions of the ball and thus the theoretical models from literature cannot be applied.

For the situations considered there were determined the flight times as mean value of the times obtained experimentally. Using the experimental data, the values of the coefficient of restitution and the losses of energy were found. One of the main conclusions is that the losses in the case of lubricated impact are greater than for the dry impact. The energy lost in the oil-wet target case is with 5% greater than for the dry target impact.

Finally, accepting for the impact force a variation similar to the ones from technical literature, an estimation of the maximum impact force is made.

References

- [1] Wang Y and Mason MT 1992 Two dimensional rigid-body collisions with friction *Journal of Applied Mechanics* **59** pp 635–642
- [2] Brach R M 1989 Rigid body collision *Journal of Applied Mechanics* **56** pp 133–138
- [3] Brach R M 1991 *Mechanical Impact Dynamics, Rigid Body Collisions* (New York Wiley)
- [4] Routh ET 1905 *Dynamics of a system of rigid bodies* (Macmillan London)
- [5] Kane T R 1985 A Dynamic Puzzle *Stanford Mechanics Alumni Club Newsletter* pp 6-10.
- [6] Goldsmith W 2001 *Impact, The Theory and Physical Behaviour of Colliding Solids* (Dover Publication)
- [7] Kilmister C W and Reeve J E 1966 *Rational Mechanics* (American Elsevier New York).
- [8] Timoshenko S P and Goodier J N 1970 *Theory of elasticity* (McGraw-Hill New York)
- [9] Lankarani H M and Nikravesh P E 1990 A contact force model with hysteresis damping for impact analysis of multibody systems *J. Mech. Design* **112** pp 369-376.
- [10] Ardema M D 2005 *Newton-Euler Dynamics* (Springer-Verlag US) p 316
- [11] Johnson K L 1985 *Contact Mechanics* (Cambridge University Press)

- [12] Flores P, Ambrosio J, Pimenta Carlo J C and Lankarani H M 2008 *Kinematics and Dynamics of Multibody Systems with Imperfect Joints, Models and Case Studies* (Springer)
- [13] Flores P, Machado M, Silva M T and Martins J M 2011 On the continuous contact force models for soft materials in multibody dynamics *Multibody Syst Dynamics* **25** pp 357-375.
- [14] Ciornei M C, Alaci Stelian, F C Ciornei and I C Romanu 2017 Use of loading-unloading compression curves in medical device design *IOP Conf Series MSE* **227** 012026
- [15] Limmahakhuna S, Oloyede A, Sitthiseripratip K, Xiao Y and Yan C 2017 3D-printed cellular structures for bone biomimetic implants *Additive Manufacturing* **15** pp 93–101
- [16] Mohamed M, Huo Z, Hawkins S, Chandrashekhara K, Birman V and Volz J 2013 Moisture effects on performance of polyurethane composite sandwich panels manufactured using VARTM, *Int SAMPE Technical Conference*, Society for the Advancement of Material and Process Engineering (SAMPE) https://scholarsmine.mst.edu/mec_aereng_facwork/698/
- [17] Kanyanta V and Ivankovic A 2010 Mechanical Characterisation of Polyurethane Elastomer for Biomedical Applications *J Mech Behavior of Biomedical Materials* **3**(1) pp 51-62